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On Ancient Eclipses. By P. H. Cowell.

One of the most striking features of Mr. Nevill's recent paper (*Monthly Notices*, lxvi. pp. 404–420) is that, whereas I maintain that the ancient eclipses can only be satisfied by one set of formulæ for the configuration of the node, Sun, and Moon, he produces four systems, and states, on p. 415, that they respectively satisfy, out of twelve eclipses, seven or possibly nine, seven or eight, six at least, but possibly eight, and ten. A moment's consideration will show that Mr. Nevill and I must be using the word "satisfy" in different senses. He is evidently more easily satisfied than I am. On p. 416 he regards as satisfactory any value of $\Delta\phi$ less than $2\frac{1}{2}^\circ$; translating this into my language he is satisfied if the tabular latitude of the Moon, corrected for the difference of parallaxes at the moment of apparent conjunction in longitude, is less than $130''$—for shortness I will say if the residual is less than $130''$. Now the whole point of my first paper on eclipses (*Monthly Notices*, lxv. pp. 861–867) is that the residuals can be simultaneously reduced to less than $50''$. Mr. Nevill therefore uses the word "satisfy" with its stringency relaxed in the proportion of 5 : 2; it is no wonder, therefore, that so vast a number of mutually exclusive systems give him this very moderate amount of satisfaction. The same considerations explain Mr. Nevill's treatment of the eclipse of Utica. On p. 867 (*Monthly Notices*, lxv.) I give as the equation arising from this eclipse

$$+23s_F - 38s_D = +39''$$

error is about $\pm 1''{\cdot}3$ for a single determination (corresponding to about $\pm 0{\cdot}03$ in the observed magnitude) and for a mean of nine determinations we have a correction $-1''{\cdot}5 \pm 0''{\cdot}5$ to the value $+4''{\cdot}1$ that has been adopted. The value deduced from lunar eclipses is therefore

$$+2''{\cdot}6 \pm 0''{\cdot}5$$

If a correction to a quantity that we know exists is obtained with a probable error one fifth part of itself, that correction is deemed well established. It is, of course, different when the existence of the quantity is in dispute. Hence it is that from the first I have admitted that the lunar eclipses are not by themselves sufficient to establish either an acceleration for the Sun or an unexpected secular acceleration for the node. But the result here obtained at least demands the examination of other evidence, such as solar eclipses, to see whether it can be confirmed.

I now come to the solar eclipses. I have first of all a few additional remarks to make about the geometry of these eclipses. In many cases the central lines, according to the present tables,

Equation of Condition.

1*a*	$+76(s_F - s_D)$	$-143 s_D =$	$-697''$	} -1062 adopted as date.
1*b*	-76	-47	$+884$	
1	-73	-60	$+29$	
2	-59	$+26$	-42	
A	-56	$+129$	$+211$	Chinese.
3	$+55$	-146	$+115$	
4	$+54$	$+98$	$+163$	
5*a*	$+52$	$+93$	-476	} -602 adopted as date.
5	$+52$	-142	$+46$	
B	-51	$+59$	$+429$	Chinese.
6	$+51$	$+72$	$+59$	} -584 adopted as date.
6*a*	-50	$+70$	$+464$	
7	-45	$+69$	$+34$	
8	$+43$	$+25$	-525	
9	$+43$	$+54$	$(+14)$	Sun already set.
10*a*	$+42$	-49	-868	} Neither date.
10*b*	$+42$	$+61$	$+178$	
11	$+40$	-19	-177	
C	-39	-44	-157	Chinese.
D	-35	$+93$	$+456$	„
E	-35	-51	$+27$	„
12	$+23$	-12	$+37$	

pass two or three hundred miles away from the place naturally indicated by the record. Now no admissible alterations of the tables—that is to say, alterations consistent with the observations of the last century—will produce changes of the required magnitude except changes to the secular terms of the elongation and argument of latitude. Hence every geometrical possibility is exhausted by assuming these two unknown quantities. On the other hand, it is wrong to assume that one of these quantities is theoretically zero, for facts must be examined first, and theories made later. If the theory is right, the facts will ultimately support it. Therefore, as long as it is a question of altering the central lines by two or three hundred miles, two unknown quantities are alone admissible. It turns out that these two unknown quantities suffice to reduce the residuals (or differences of latitude at apparent conjunction in longitude) to less than 50″. Further than this I cannot go, for directly it becomes a question of small corrections there is a large choice of possible alterations to the tables competent to produce them. The mean motions may be altered by 10″ or 15″ a century, but not more; the secular term in the lunar perigee may need a small correction; and of course supplementary small corrections may be given to the two principal unknown quantities.

Any formula put forward as satisfying ancient eclipses will still satisfy them if modified by the addition of

$$\lambda(25T + T^2)$$

for this quantity is not large for the epochs of the eclipses. Modern observations will ultimately settle the values of λ for the two formulæ for D and F respectively; all that can be said at present is that λ is less than [illegible]″·5. This consideration will not

Full Calculations for the Eclipse of −708 *July* 17 *at Heeng-yang.*

$$T = -25{\cdot}074254 \qquad T^2 = 628{\cdot}72 \qquad T^3 = -15765$$

Precept.	g	ω	$-\Omega$	L′	π'
T × last 4 figs. of centennial motion ...	− 145 280″	− 38 013″	− 73 242″	− 69 355″	− 155 109″
T × rest of centennial motion	−43075 311 689	−541 854 629	−174 516 808	− 153°·144	...
Integral number of revolutions	+43075 152 000	+541 728 000	+174 960 000	...	...
Secular term ...	+ 31 310	− 28 544	− 4 778	+ 3 269″	+ 1 006
Cube term ...	− 788	+ 694	+ 110	...	− 189
Sum	− 76° 14′ 7″	− 53° 28′ 12″	+101° 28′ 2″	−171° 30′ 4″	− 42° 51′ 32″
Add value at 1800·0 ...	34° 5′ 31″	138° 39′ 13″	68° 11′ 41″	108° 24′ 25″	236° 38′ 15″

diminish the slight discordance that there is between the mean result of the lunar eclipses and the result of the solar eclipses, but may possibly diminish the difficulty due to the observations from 1750 to 1800.

The value of the Sun observations in their present state for the period 1750 to 1800 is not great ; possibly they are capable of improvement on re-reduction ; at present, however, they fail to stand the only test that I can apply : they give (see Newcomb's Astronomical Constants, p. 22) a wholly impossible mass of *Venus*, although the period of the *Venus* perturbations is such as to put systematic errors out of the question. The systematic mean error for the same period may easily be large. How accurately could the clocks of those days carry on the clock error from noon to the mean clock star at about 10 P.M. ? How large was the diurnal inequality in level before it was recognised as necessary to keep the shutters closed in the daytime ? Again,

Hence $F = 172° 44' 44''$, $L = 104° 33' 3''$, $D = 356° 8' 38''$ $-g' = 128° 13' 50''$.

Inequalities of Longitude.

Argument. Symbolic.	Argument. Numerical.	Coefficient.	Inequality from MS. Table.
	°	''	''
g	34·092	22640	+12 690
$-g+2D$	318·20	4586	− 3 056
$2D$	352·29	2370	− 318
$2g$	68·2	769	+ 714
$-g'$	128·2	669	+ 526
$-2F$	14·5	412	+ 104
$-2g+2D$	284·1	212	− 206
$-g-g'+2D$	86·4	206	+ 206
$g+2D$	26·4	192	+ 85
$-g'+2D$	120·5	165	+ 142
$g-g'$	162·3	148	+ 45
$-D$	3·9	125	+ 9
$-g-g'$	94·1	110	+ 110
$-2F+2D$	7	55	+ 7
$-g-2F$	340	45	− 15
$g-2F$	49	40	+ 30
$-g+4D$	310	38	− 29
$3g$	102	36	+ 35
$-2g+4D$	276	31	− 31
$g-g'-2D$	170	29	+ 5
$-g'-2D$	136	24	+ 17
		Sum =	+11 070

are we entitled to say that there are no unknown long-period terms in the motion of the Sun when we know that such terms exist in the case of the Moon? For these reasons I prefer the hypothesis of a secular acceleration of the Sun to that of a secular acceleration of the node contrary to gravitational theory.

In exhibiting the equations of condition for the solar eclipses in my first paper I gave linear equations between s_F and s_D, the corrections required by the secular terms of the argument of latitude and the elongation. Since working up the lunar eclipses it is preferable to take $s_F - s_D$ and s_D as the two fundamental quantities, because these are the quantities determined by the magnitudes and times of the lunar eclipses respectively. The coefficient of $s_F - s_D$ is $\pm 0{\cdot}0895$ T^2, for the ascending or descending node respectively, and the coefficient of s_D is $(\pm 0{\cdot}0895 - k)T^2$.

Inequalities of Latitude.

Argument. Symbolic.	Argument. Numerical.	Coefficient.	Inequality from MS. Table.
F	172°746	18461″	+ 2 331″
$g+F$	206·84	1010	− 456
$g-F$	221·34	1000	− 660
$-F+2D$	179·6	624	+ 4
$-g+F+2D$	130·9	199	+ 150
$-g-F+2D$	145·5	167	+ 94
$F+2D$	165·0	117	+ 30
$2g+F$	241	62	− 54
$g-F+2D$	214	33	− 18
$2g-F$	255	32	− 31
$-g'-F+2D$	308	30	− 24
$-2g-F+2D$	111	16	+ 15
$g+F+2D$	199	15	− 5
$-g'+F-2D$	309	12	− 9
		Sum =	+ 1 367

Parallax &c. (for numerical arguments, see inequalities of longitude).

$$p = 3423'' + 187'' \cos g + 34'' \cos(2D - g) + 28'' \cos 2D + 10'' \cos 2g$$

$$= 3423 + 155 + 25 + 28 + 4 = 3635''$$

$$p - p' = 3626''$$

$$\frac{d}{dt}(V - V') = 1632'' + 227'' \cos g + 13'' \cos 2g - 5 \cos g'$$

$$= 1632 + 188 + 5 + 3 = 1828''$$

$$-\frac{dU}{dt} = 163'' + 20'' \cos g + 2'' \cos 2g = 163 + 17 + 1 = 181''$$

Equation of centre of Sun.

$$e_1' = 7324'' \sin g' = -7855 \text{ product } -5753''$$
$$e_2' = 82 \sin 2g' = +97 \quad ,, \quad +80$$
$$\text{Sum } -5673$$
$$V - V' = +2861$$

$\sin \varepsilon = 4035$ $\cos \varepsilon = 9151$

Latitude $\lambda = 34^\circ\ 12'$ $\sin \lambda = 5621$ $\cos \lambda = 8271$

G.M.T. $314^\circ{\cdot}154$ Longitude $109^\circ{\cdot}050$

Local Mean Solar Time $63^\circ{\cdot}204$ $L' = 108^\circ{\cdot}407$

Local Sidereal Time h $171{\cdot}611$ $V' = 106{\cdot}831$

$h - V' = 64^\circ{\cdot}780$ $h + V' = 278^\circ{\cdot}442$

Parallactic Displacements.

$(p-p') \sin \lambda \sin \varepsilon$ 822 ; $\cos V'$ -290 product -238

$(p-p') \cos \lambda \cos^2 \frac{\varepsilon}{2}$ 2872 ; $\sin (h-V') + 9047$,, $+2598$

$(p-p') \cos \lambda \sin^2 \frac{\varepsilon}{2}$ 127 ; $-\sin (h+V') + 989$,, $+126$

$$v' = \text{sum} = +2486''$$

$(p-p') \sin \lambda \cos \varepsilon$ $= 1865$

$(p-p') \cos \lambda \sin \varepsilon$ 1210 ; $-\sin h - 1459$ product -177

$$u' = \text{sum} = +1688''$$

$0{\cdot}2295 \times (p-p') \cos \lambda \cos^2 \frac{\varepsilon}{2}$ 659; $\cos (h-V') + 426$ product $+281$

$0{\cdot}2307 \times (p-p') \cos \lambda \sin^2 \frac{\varepsilon}{2}$ 29 ; $-\cos (h+V') - 15$,, -4

$$\frac{dv'}{dt} = \text{sum} = +277''$$

$0{\cdot}2301 \times (p-p') \cos \lambda \sin \varepsilon$ 278; $-\cos h + 989$

$$\frac{du'}{dt} = \text{product} = +275''$$

Hence

$$V - V' - v' = +375'' \qquad U - u' = -321''$$
$$\frac{d}{dt}(V - V' - v') = +1551'' \qquad \frac{d}{dt}(U - u') = -456''$$
$$k = -0{\cdot}294$$
$$\text{Residual } = k(V - V' - v') - (U - u') = +211''$$

The Eclipse at Babylon.—Equations of condition are given for the eclipses of −1123, −1116, and −1062; and in *Monthly Notices*, lxv. p. 867, the equation of condition for −1069 with slightly different formulæ is seen to be

$$74(s_F - s_D) - 189 s_D = +655''$$

From these alternatives I select −1062 as the date of the eclipse. As to the meaning of Sivan, the equation

$$26 \text{ Sivan} = \text{July } 31$$

must be accepted, or the record must not be interpreted to mean that there was a total solar eclipse at Babylon.

The Eclipse of Nineveh.—In addition to having been recorded at Nineveh, it is clear that this eclipse was seen by the prophet Amos (see Amos viii. 9).

I have taken as the place for calculation lat. 32° 40′, long. 35° 21′.

Calculating as in other cases, I obtain

$$\begin{array}{llll}
v' = +\ 198 + \ 37 - 62 & = +173'' & \mathrm{V} - \mathrm{V}' - v & = +1109'' \\
u' = +1778 - 1187 & = +591 & \mathrm{U} - u' & = +\ 152 \\
\dfrac{dv'}{dt} = +\ 666 + \ 26 & = +692 & \dfrac{d}{dt}(\mathrm{V} - \mathrm{V}' - v') & = +1112 \\
\dfrac{du'}{dt} & = -\ 68 & \dfrac{d}{dt}(\mathrm{U} - u') & = -\ 110
\end{array}$$

Hence $k = -0''{\cdot}100$, and the residual is $-263''$.

The central line therefore passed a long way to the north of Samaria according to my formulæ.

The Chinese Eclipses.—Five eclipses have been calculated for Heeng-yang. The equations of condition are

$$\begin{aligned}
-56\ (s_F - s_D) + 129\ s_D &= +211'' \\
-51\ (s_F - s_D) + \ 59\ s_D &= +429 \\
-39\ (s_F - s_D) - \ 44\ s_D &= -157 \\
-35\ (s_F - s_D) + \ 93\ s_D &= +456 \\
-35\ (s_F - s_D) - \ 51\ s_D &= +\ 27
\end{aligned}$$

I cannot find any alteration of the formulæ or of the place of observation that will make these five eclipses all total at the same place. Our information about them is evidently deficient. They appear to be valueless, and they neither support nor conflict with any particular formulæ.

The Eclipse of Esar-haddon.—The record is perhaps satisfied if the central line intersects the line of communications between Nineveh and the Assyrian king who had gone to Egypt. The positive residual $+115''$ given by my formulæ for Nineveh shows that the central line passed south of Nineveh. I therefore do not think that this eclipse conflicts in any way with my formulæ.

The Eclipse of Susa.—This eclipse is considered by Mr. Nevill in *M.N.* lxvi. p. 411; it is not, however, quite apparent why he has labelled the eclipse "of Susa," for I am informed that the tablet was found at Nineveh; and the words quoted by Mr. Nevill contain no reference to Susa. I have, however, calculated for Susa, and I find that the central line passed considerably to the south. The record is very vague, and the details unintelligible. The Sun, as Mr. Nevill points out, cannot be troubled for three days in an eclipse.

The Eclipse of Archilochus.—My formulæ supply a reasonable explanation of how it was that Archilochus saw a total eclipse. On the other hand, many other formulæ would do as much. Mr. Nevill says we have a choice of seven eclipses. Professor Millosevich excludes three of these and leaves four. In addition Archilochus is known to have lived in Paros as well as Thasos. I therefore withdraw the eclipse of Archilochus, merely congratulating myself that my formulæ do not compel me to suppose that by some curious coincidence the eclipse took place just as Archilochus had got half-way between Paros and Thasos.

The Eclipse of Larissa.—Equations of condition are given for -609 and -602. The latter date fits in with my formulæ.

Mr. Nevill's historical note on this eclipse is most valuable.

The Eclipse of Thales.—Equations of condition are given for -584 and -556; for the former date my formulæ make the central line pass very slightly to the south of Iconium at about $7^h\ 4^m$, or about three minutes before sunset. There is of course an uncertainty as to the exact position of the battlefield; but this is compensated by the fact that formulæ which throw the central line much further south make it miss the mainland altogether.

If an eclipse be total just before sunset, the parts of the Sun that are first uncovered by the Moon are roughly the same as those that first disappear below the horizon. The phenomenon may thus be somewhat prolonged and to that extent more terrifying.

The Eclipse of Thucydides.—The careful language of Thucydides may advantageously be compared with his description of a partial eclipse that occurred seven years later (Thuc. iv. 52). Thucydides clearly does not describe partial eclipses as if they were total or annular. But in any case Thucydides would never have imagined the appearance of certain of the stars. He saw it. Now in *A.N.* 3682 we read that the path of this eclipse

"did not come within 500 miles of Greece." These 500 miles are, I believe, to be measured in a north-easterly direction. What does Professor Newcomb think that Thucydides was doing in the middle of the Black Sea? It is reasonable to impose the condition that Thucydides was not at any rate further off than the Bosphorus, and this limitation, to anyone who believes that the present tables correctly represent the positions of the Sun and the node, will necessitate the reduction of the secular acceleration of the Moon to its theoretical value. I however see no reason to doubt that Thucydides was in Athens, as my formulæ would make him. We know he was in Athens the following summer, when he caught the plague. He tells us on another occasion when he went on foreign service; his silence presumes that he was not on foreign service in the first year of the war. But he might, it is said, have gone to look after his estates in Thrace. If so, I do not think that he would have been so careful to explain that he was aware at this very time of the exceptional importance of the war.

"Thucydides, an Athenian, wrote the history of the war in which the Peloponnesians and the Athenians fought against one another. He began to write when they first took up arms, believing that it would be great and memorable above any previous war; for he argued that both States were then at the full height of their military power, and he saw the rest of the Hellenes either siding or intending to side with one or other of them. No movement ever stirred Hellas more deeply than this; it was shared by many of the Barbarians, and might be said even to affect the world at large." (Jowett.)

And, again, if he saw stars in the North Ægean, and if stars were not seen at Athens, he would very likely have known it.

The Eclipse of Lysander.—Perhaps no other eclipse is alluded to more slightly than this. The eclipse of −762 at Nineveh is recorded briefly, but it is recorded for its own sake. The eclipse of Lysander is recorded simply to mark the time of some events in Thrace. Fighting took place περὶ ἡλίου ἔκλειψιν. Following Mr. Nevill, I have calculated this eclipse for Athens, though I do not understand why Thrace should not be taken in preference to Athens as the place of observation. But in any case the eclipse was partial, the central line falling further north.

The Eclipse of Ennius.—To make this eclipse total before sunset, the secular acceleration must be increased largely. The record, however, does not allege totality, and in any case the authority is very slight. Cicero lived 150 years after Ennius, and Ennius lived 200 years after the eclipse.

The Eclipse of Pelopidas.—The eclipse of −363 was certainly only partial at Thebes. Oppolzer's diagrams suggested the eclipse of −360 to me, but that does not fit in either. Moreover, it would perhaps have been in conflict with current chronology that fixes the earlier date for the death of Pelopidas. The authority is Plutarch, who lived some 400 years later than the events he describes.

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Moreover, the same accuracy is not to be expected of an historian who ranges over so vast a field as Plutarch as from a specialist who writes about a shorter period. On the other hand, Plutarch fills in details. He describes not only the eclipse, but the effect produced by the darkness.

The Eclipse of Agathocles.—In a footnote in *Monthly Notices*, lxvi. p. 417, Mr. Nevill points out a distinction between my two hypotheses: (i.) assigning a secular acceleration to the node different from that required by theory; (ii.) assigning a secular acceleration to the Sun. It is perfectly true that at the same tabular time the two values of the difference of longitude will differ roughly by

$$0''\cdot4\ T^2 \cos g$$

In forming the residual, however, this quantity has to be multiplied by "*k*." The product is often negligible, but in this case it happens that it is as much as 40″. My first hypothesis, therefore, requires that Agathocles should have got to a distance of at least eighty miles from Syracuse, and my second to a distance of at least 120 miles. Both distances are perfectly possible. The account says that he only got away from the Carthaginians after dark. He was therefore in full flight for a whole day—possibly fifteen hours. Grote states that "during the night he made considerable way." In *Nature* (1906 November 2) I have discussed the historical evidence. Agathocles appears to have left Syracuse early on August 14 with a favourable wind, and there are many indications that he went northwards.

The Eclipse of Utica.—My formulæ make the eclipse of +197 total at Utica, and supply a commentary upon the words of Tertullian that does not lose its force even if it be proved that Tertullian wrote twelve years later.

Professor Newcomb says that I ignore Tertullian's "pœne." I regard my eclipse papers as proving empirical corrections removing errors from the present tables of the order $4''\ T^2$. As we cannot replace $4''\ T^2$ in the formulæ by $-100''\ T$, because such a correction is incompatible with modern observations, it follows that there is no geometrical alternative to correcting the secular terms until the errors of the present tables have been reduced from the order $4''\ T^2$ to, say, the order $0''\cdot4\ T^2$. When this degree of accuracy is reached small corrections may be applied, either to the secular terms or mean motions; in fact, we get as many arbitrary quantities as equations of condition; and hence I consider that my formulæ will only give eclipse tracks correctly to within fifty miles. I maintain that they are free from errors of 200 to 300 miles, and that the present tables are not free from such errors. Therefore, if Professor Newcomb likes, he may take the word "pœne" to mean that the central line missed Utica by a few miles. There is the further obvious alternative that "pœne" merely corresponds with the fact that there is not black darkness even on the central line of a total eclipse.

I understand Professor Newcomb to suggest (*Monthly Notices*, lxvi. p. 470) that because Airy and others deduced erroneous conclusions from ancient eclipses, therefore my conclusions will, in all probability, turn out also to be unsound. I will therefore conclude by examining the basis of Airy's conclusions. His last paper, entirely contradicting his previous paper in the *Transactions of the Royal Society* for 1853, is published in the *Memoirs, R.A.S.*, vol. xxvi. Inasmuch as his conclusion is that Hansen's tables represent the ancient eclipses that he examined, he has this in common with me, that his formulæ, as well as mine, will satisfy a century of modern observations; the same cannot be said of Oppolzer and Ginzel. Airy, however, only uses three eclipses. One of these three is the eclipse of Agathocles, where the uncertainty in the position of Agathocles causes the eclipse to satisfy nearly any formulæ; another eclipse is that of Thales, which is well known to be satisfied by Hansen's tables; and the third eclipse is that of Larissa, to which Airy assigns an impossible date. Inferences from eclipses, in view of the want of absolute precision in the records, depend upon the production of an overwhelming degree of coincidence. Airy's investigations, in reality, deduce a result from Thales, and show, as they were nearly certain to do, that the central line for Agathocles comes within one day's journey of Syracuse; and the whole degree of confirmation lies in the fact of the supposed agreement of the eclipse of Larissa, for which he has taken a wrong date. In no previous attempt to explain ancient eclipses has it ever happened that confirmation has been obtained from other eclipses. But in my case the eclipse at Babylon, the lunar eclipses, the eclipses of Thales and Larissa have all supported the explanation at which I had previously arrived from other evidence, viz. the eclipses of Nineveh, Archilochus, Thucydides, Agathocles, and Tertullian.

By what right is Airy taken as the basis for an argument that, if Airy was wrong, others too will probably be wrong? What portion of Airy's "grand labours" on the Moon, to quote a recent phrase of Lord Crawford's, survives to the present day? Is there any correct result relating to the Moon connected with his name? His measurement of the constants is most inaccurate. With a hundred years' observations at his disposal, he failed to detect any one of three short-period errors with coefficients exceeding 3″ in his tabular longitudes; his analysis is based upon the two erroneous assumptions that his adopted solar terms are right, and that the errors of his tables depend on the position of the node. On both these points it was Mr. Nevill who got into the right path: it was he who first showed that the eccentricity of the Moon could not be determined correctly as long as certain errors in the solar terms were ignored (*Mem. R.A.S.*, vol. xlviii. pp. 315, 318), and it was Mr. Nevill who first showed that the defect of the tables was largely due to the short-period perturbations produced by the planets.